The Niche for Educational Software in Mathematical Learning

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Abstract

The application of educational software in mathematical learning is a challenging goal. Unfortunately, many attempts to apply modern technology in the conventional learning process fail to reach the level of benefit which is expected. This paper discusses the main factors which determine the success of educational software and provides information of one specific series of educational modules build in Elica. These modules are based on the concepts of simplicity, uniqueness and attractiveness.

Keywords

Educational software, mathematical learning, 3D modeling, DALEST, Elica

1. Introduction

Mathematical learning is an important phase of the life of every student. It is not related to only learning Mathematics, but also applying ideas and thinking patterns developed in Math classes onto other domains. The significance of the mathematical learning is that it provides a mechanism for people to analyze things logically and to investigate problems in a systematic way.

The process of learning is neither smooth, not conventional. Educators are still looking for new tools and ideas which will make the learning process a more successful one. Part of the efforts is dedicated to the use of modern technology, mainly computers and software. It is believed that Mathematics is the place where educational software will easily find a niche. As it is believed the core of Mathematics is doing calculations and computers are all about calculations, so it sounds reasonable to use computers in Mathematics. However, calculus is just one piece of the Mathematics, and using computers to calculate is a bad use of the modern computers. As a result of not understanding this a lot of mathematical educational software has been developed trying to master mathematical skills which can be mastered well without computers.

2. The DALEST Project

2.1. First phases of DALEST

To address the issue of effectiveness of computers in education, the DALEST project aims at developing learning environments which will use educational software in the way that it complements the traditional tools and means of learning. The partners of the DALEST (*"Developing an Active Learning Environment for Stereometry")* project are are University of Cyprus, University of Southampton, University of Lisbon, University of Sofia, University of Athens, N.K.M. Netmasters and Cyprus Mathematics Teachers Association. The project is co-funded by the European Union under the Socrates Program MINERVA, 2005 Selection [1].

The first phase of the project was to study and compare the existing carricula in three-dimensional geometry in Mathematics for grades 5-8 across several European countries: Bulgaria, Cyprus, England Greece and Portugal. The next phase was to design and implement educational software which can be used for three-dimensional geometry and Stereometry to deal with the solids and the measurements of volumes and surfaces of various figures like spheres, cones, cylinders, prisms, pyramids.

2.2. DALEST/Elica subproject

The design and development efforts were split into several subprojects in attempt to cover different view points. One of the subprojects was to build applications based on already existing educational software, namely Elica – Educational Logo Interface for Creative Activities [2]. The reason to use Elica is that it is suited for building such types of applications:

- Elica is a general programming environment and it is not fixed to a set of predetermined topics. It is flexible and allows the building of various modules.
- The underlying programming language is a dialect of Logo. Logo is the mostly used educational programming language for the last decades.
- Elica has 3D graphics, a library of 3D solids, and programmable interactivity.
- Elica has already been used for creating educational software.
- Elica is free.

2.3. Preexisting Elica educational applications

Elica by itself is an educational application, but it is used as a meta tool to build other, higher-level applications. There were several Elica-based applications build before the start of the DALEST/Elica project. Some of them are microworlds that extend the functionality of Elica and are used in a programming style. Others have graphical interfaces and users work with them in a point-and-click style.

Equation Balance [3] is an application for solving linear equations with integer coefficients and integer solutions. It represents the equation as a balance. There are two types of object to manipulate. The small golden bars represent the known values in the equation. The boxes represent the unknown values. Figure 1 shows a snapshot of the playing panel of

Figure 1. A snapshot from the Equation Balance application while solving the equation 8x+7 = 31

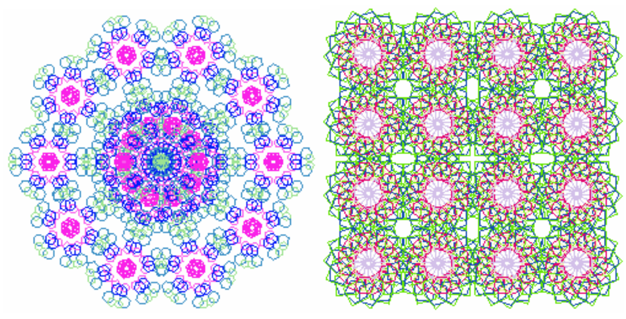

Figure 2. Two of the patterns generated by the Pattern Constructor application.

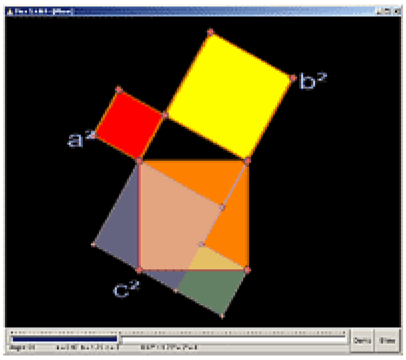

Figure 3. A dynamic proof of the famous Pythagoras Theorem

Equation Balance. The equation which is under “investigation” is 8x+7=31.

Students can modify the objects on the plates in order to find out how many golden bars are there in a single box. While doing this they maintain the equilibrium of the balance by removing the same amount of weight from both plates. Students can add weight too (both as bars and boxes) and thus they can create their own equations.

Pattern Constructor [4] is an application for pupils. It starts with polygons that can be flipped, translated and rotated. The new polygons are blended with the old ones producing nice patters (Figure 2). Transformations are stored in a stack and pupils can pop them out to undo changes.

Pythagoras Theorem [5] implements one of the hundreds proofs of this famous theorem. There is a slide bar at the bottom of the window – see Figure 3. When the thumb is moved, the figure on the screen changes, but the construction can still be used to find the proof in the same way. An interactive tool like this helps students to explore variations of the same problem, thus filtering out the variant from the invariant part of the proof.

The North Pole [6] project is inspired by the first flight over the North Pole in 1926, by Byrd and Benet [7]. This project is implemented as a mini-microworld. It helps student to find out the trajectory of the plane flying at various speeds. The flight is observed from above the Earth, thus the planet rotation is taken into account. Figure 4 shows just few of the many possible trajectories. Studying the relation between the speed of the airplane and the shape of the trajectory is a nice and challenging mathematical exercise [8].

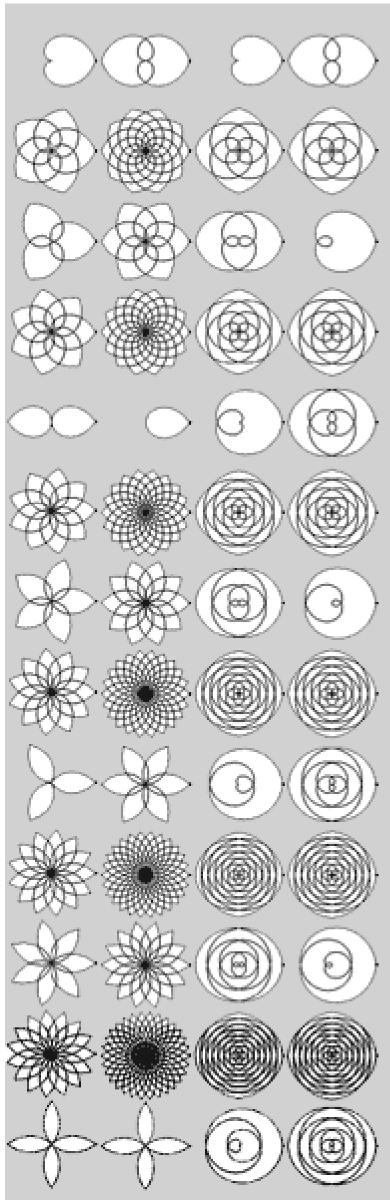

Figure 4. Some of the possible trajectories generated by the North Pole application

Geomland is a geometry-oriented Elica Logo library that enhances Elica core Logo. It is a microworld that helps students and teachers to explore various plane geometry problems by programming constructions.

This is an example of an Elica Geomland program:

```
make "scale 0.3
ob "O point 0 0
ob "k circle point 0 0 100
ob "a 0
ob "r (exp (cos :a)) - (2 * (cos
       (4*:a))) + (sin (:a/12))^5
ob "P pointon (line :O :a) :r
ob "H locus "P "a 2 180
ob "Butterfly polygon :H
```

It draws a nicely shaped butterfly – see Figure 5, by defining a parametric equation of the contour.

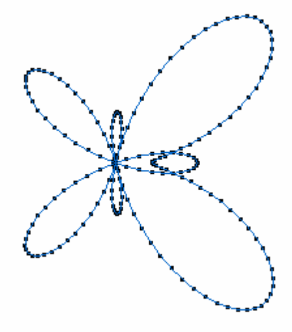

Figure 5. Mathematical butterfly drawn by a Geomland program.

3. DALEST/Elica Projects

Most of the pre-DALEST Elica-based applications were two-dimensional. It is so because when software is used for mathematical learning in most cases only two dimensions are considered. However, the real strength of technology enhanced education is in areas which go beyond the flatness of the plane. The ultimate goal of the DALEST project is to reach the three-dimensional domain of Stereometry. This poses several specific requirements to the design of the Elica-based applications.

3.1. Requirements

Building an application can follow two different paths. The first is to build it and then look for possible applications. The other is to first identify possible applications and then build it. The design of DALEST/Elica applications follows the second path by providing a set of independent applications, rather than one single multifunctional one.

An important role in the design process was to enumerate and consider all factors which determine what application is acceptable:

- *Uniqueness.* The applications must practice activity which is not typically covered by any other application. It does not make sense to build yet another application doing the same old stuff.
- *Applicability*. There are many activities which can be recreated in a software environment. However, some of them should not be computerized, because this will not increase the understanding. The applicability of an application is measured with the degree of how difficult it is to do the activity without a computer. Activities which can be easily done with pencil, paper and scissors are not considered applicable for the DALEST/Elica subproject.
- *Attractiveness.* Most of the today's mathematical applications focus on functionality. Considering the fact that the Elica users will be middle-school students, an important factor in the design is whether an application can be made visually attractive without sacrificing functionality.

3.2. Activities

After searching for possible Elica-based activities a list of activities has been identified. They can be classified in several groups:

- *Activities with volumes*. Examples of such activities are the design a bottles of various shapes and fixed capacity; solving problems for pouring water between spherical, conical and cubical containers; measuring volumes of solids by submerging them in a larger vessel full of water;
- *Activities with shapes*. Such activities rely on recognizing the solids in a group by their shadow; or by their intersection with a plane.
- *Activities with unit-size cubes*. The core of these activities is a structure built up by unit-size cubes. The volume and the surface of a structure can be measured or calculated by investigating the structure or its projections.
- *Activities with nets*. Paring folded and unfolded nets; unfolding a cube into a given net.

The list of activities will grow as new ideas come to consideration. At the time of preparing this paper, there are four applications which are already developed.

3.3. The Cubix application

This is the first DALEST/Elica application. There is a structure made of unit-size cubes – Figure 6. Users have to calculate the volume and the surface of the structure. To find the volume they have to count the number of cubes in the structure, but in some

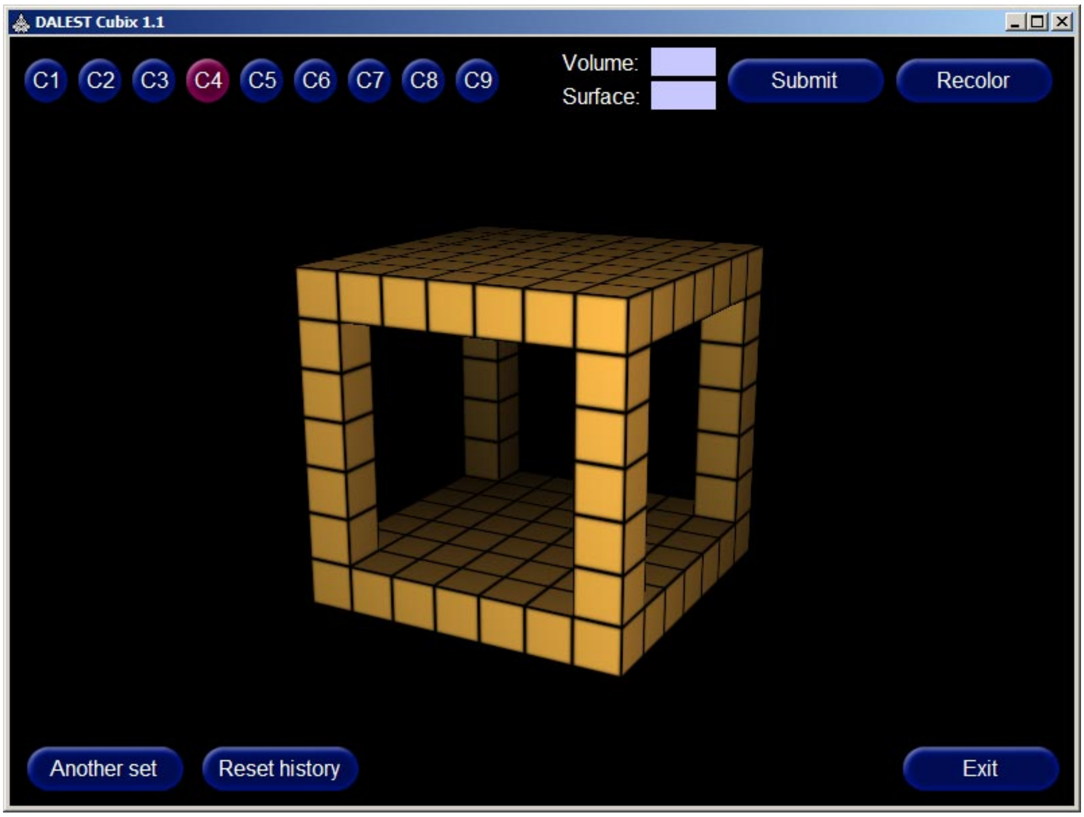

Figure 6. Problem C4 from the Cubix application.

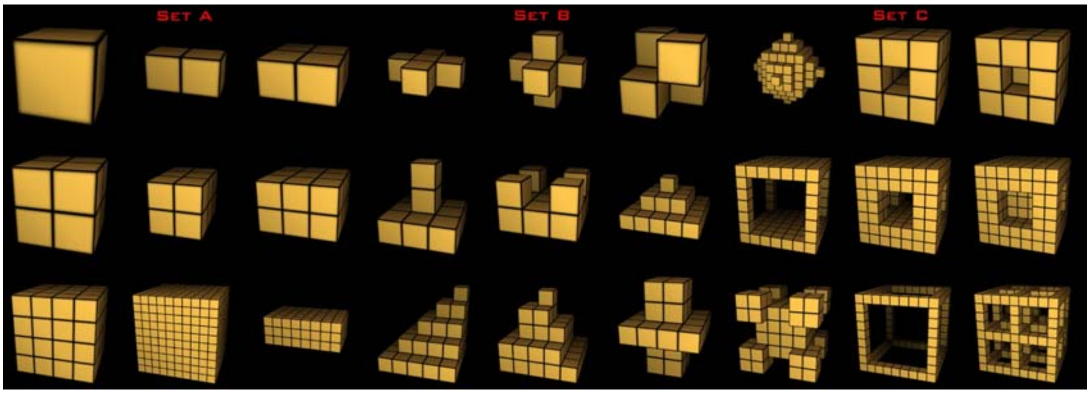

Figure 7. Figures used in the Cubix application.

problems counting is impossible or it is quite boring. In such cases students have to dividing the structure into smaller ones or remove blocks from a larger, but more regular structure until they get the current structure. Some of the figures have tunnels, other have niches. If students need they can rotate each of the figures and look it from different points of view. Figure 7 shows the structures used in the first version of Cubix.

3.4. Slider application

Imagine a magic cube with a 3D solid inside it. The solid is invisible until a laser plane-beam passes through it. This is the topic of the Slider application. The only visible part of the invisible solid is the contour of the intersection with the laser plane – see Figure 8. By moving the plane forward-backward and investigating how the intersection changes the student is supposed to find out what the mystery solid is. Because solids

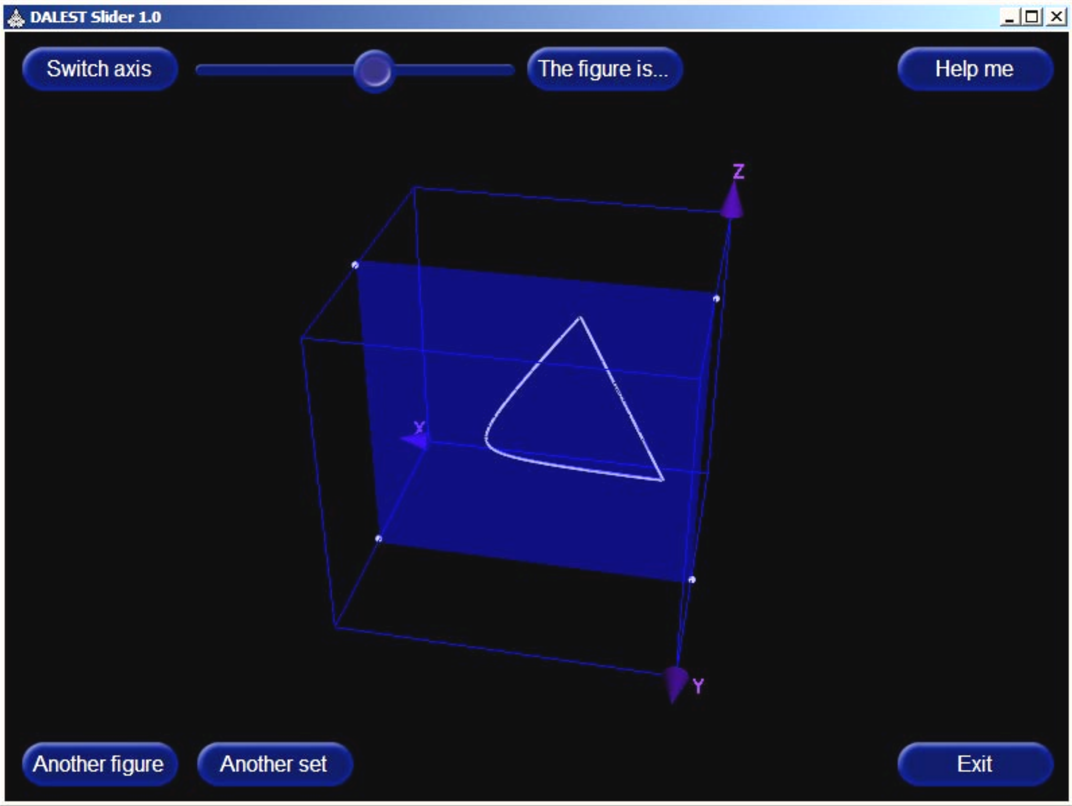

Figure 8. The laser plane illuminates a slice of a solid.

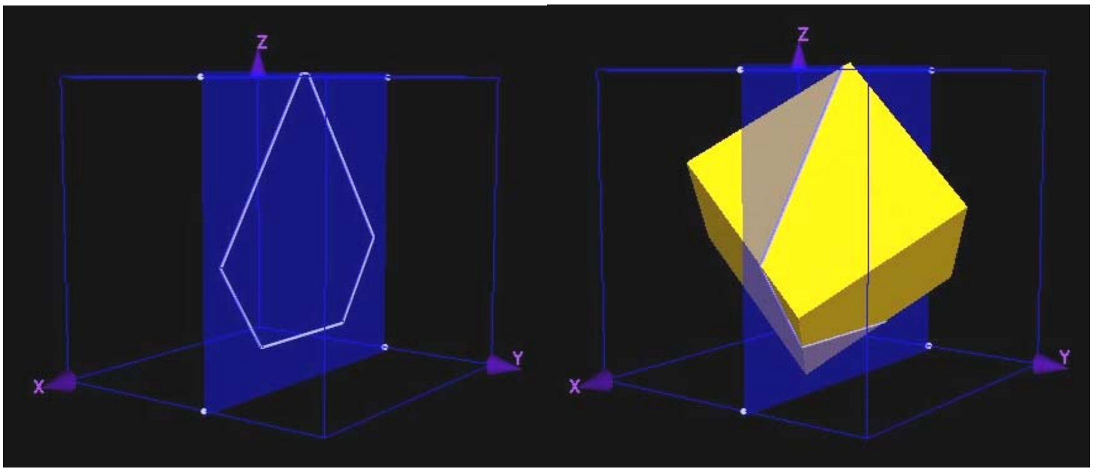

Figure 9. One of the unexpected intersections of a cube

can be rotated in any way, the laser plane could move along any of the cube's sides, thus revealing different intersections. For example when the plane is moving up-down and the intersection is a circle, but the side-view intersection is a rectangle, then the solid is either a cone or a cylinder. The exact answer depends on whether the radius of the circle changes.

The Slider application can be used to study the curves – parabolas, hyperbolas and ellipses. The intersection in Figure 8 suggests a cone. Other intersections, like the one in Figure 9, are more confusing for a first-timer. When students cannot identify the solid they can ask the application to show the solid. When the object becomes visible they can play again with the plane to see why the intersection has one or another shape.

The Slider application comes with more than 90 problems grouped in 5 problem sets. The easy set uses objects in their default orientation. More advanced sets are based on solids with different orientations in the space, thus producing various intersections. There is also a set for conic intersections where objects are cylinders and cones.

3.5. Stuffed Toys application

As it was mentioned in section 3.2, there are some activities with cube nets which are considered suitable for Elica. Folding a net into a cube and finding all possible nets that fold into a cube [9] are problems which has quite a lot implementations. Nowadays there are many on-line applets or just animations which show how to do this. The factor of availability is also negative because it is easy to make and fold nets by using paper and scissors.

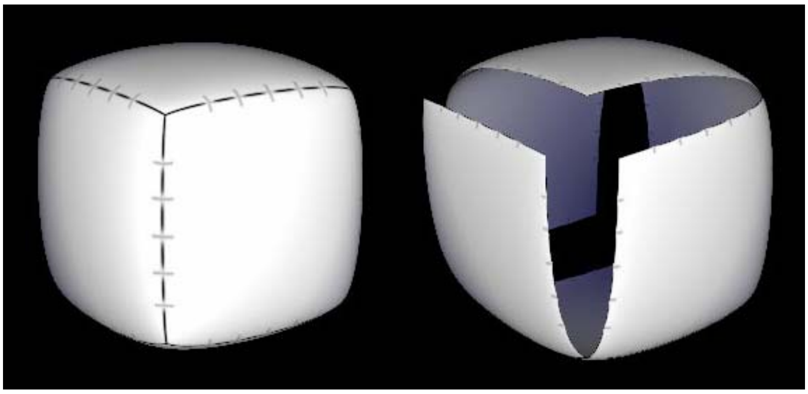

Figure 10. Stuffed cube before and after the rip off.

It is hard to think of any activity with nets, which will fulfil all three citeria. This is the reason to decide and implement the reversed problem of cube folding.

The Stuffed Toys application presents to the user a cube which looks like a stuffed toy – see Figure 10. Then the cube is ripped off in a random way so that after unfolding it could become a planar net. The role of the user is to identify which of the possible 11 unfoldings will occur. The main difficulty to the user is the fact that the ripped toy can be rotated, but cannot be unfolded. Students must unfold it in their minds and this activity is a rewarding challenge.

Except for regular stuffed cubes the application can work with other forms, which are topologically equivalent to cubes – they are composed of six faces connected the same way as in cubes.

Figure 11 shows several of the stuffed toys in their original and ripped off state. It has been found that the curvature of the faces gives additional difficulty to the recognition of the net. It becomes harder to unfold the toy in one's thoughts.

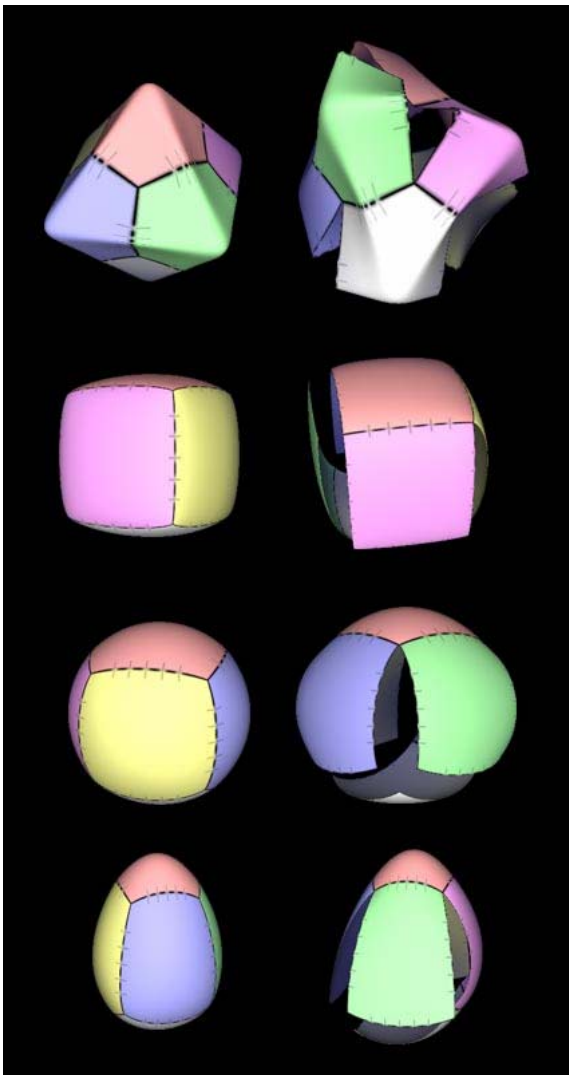

Figure 11. A few of the other stuffed toys – an octahedron, a cube, a ball and an egg. Each of them is equivalent to a cube and unfolds into one of the cube's nets.

3.6. Scissors application

Using the code of the Stuffed Toys it was possible to develop another interesting application, called Scissors. It implements the true reverse problem of cube folding. The system provides a stuffed cube and the student decides which seams to cut in order to reproduce a given net after unfolding.

Decision where to cut appears to be even harder and more challenging compared to the Stuffed Toys problems. Students must pay attention to several things. If they cut more than the required number of seams the ripped cube cannot be unfolded completely, because some faces will be entirely disembodies. If students cut

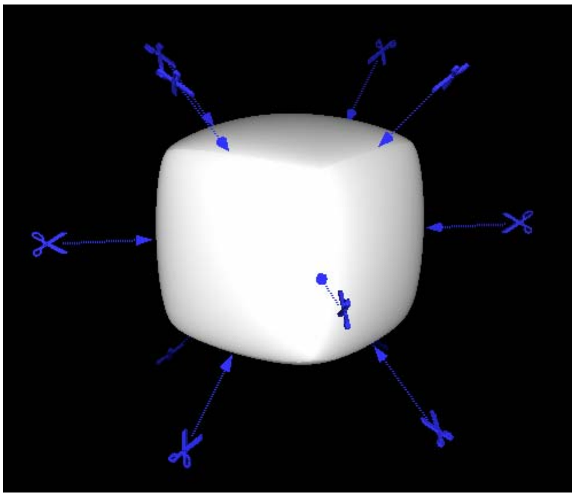

Figure 12. A stuffed cube surrounded by 12 scissors ready to cut it along its seams.

less seams than needed, the resulting figure could not be unfolded into a *planar* net. And finally, even if the number of ripped off seams is the correct one, the faces could unfold into another net, not the one which is desired.

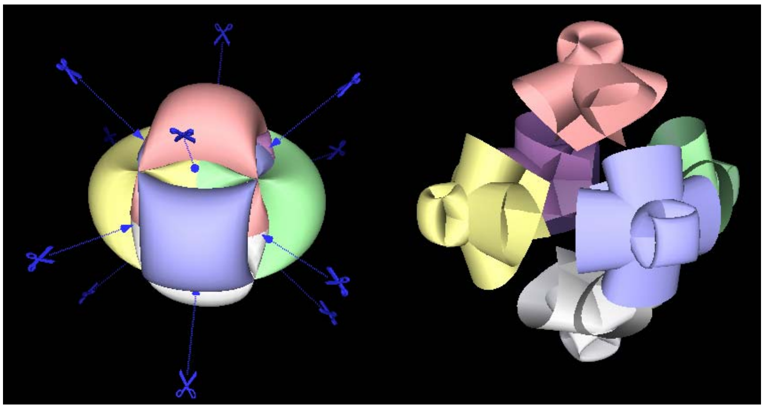

Figure 13. The Ribbons set. On the left is a complex structure made of six differently coloured bands. On the right is the same structure, but all bands are separated into individual ribbons.

There is no any simple strategy to decide what seams to cut. Students have to think several steps ahead. At the same time they have to maintain an imaginary cube in their thoughts.

After playing with Scissors for some time students will develop necessary skills and imagination to cut a cube in any given way.

But this is not the top level in the Scissors. There is yet another set of extremely complex objects which require even higher level of abstraction. These are the ribbon problems.

Imagine a set of six ribbons as shown on the right half of Figure 13. Then the bands are stitched up in to what is seen on the left of the same figure. The problem to cut off only some of the seams is much harder, because the visual image does not provide enough intuitive feedback and students might not be able to interpret the situation correctly, unless they have developed a high level of abstraction which drives them far beyond the simple stuffed cube.

4. Summary

Mathematical learning is a focal point for many educators and researchers. They do their best to design and implement computer software which will help to better understand the Mathematics behind the scene of everyday life. Special attention is devoted to developing skill in areas like three-dimensional geometry, because this is a domain where ordinary textbooks cannot provide effective environment for learning.

Educational software tries to overcome the limitations of static and plane representation of space and solids, thus becoming an adequate tool which can be used in middle schools. The DALEST project combines the challenges of developing modern educational software with the challenges of the understanding Stereometry. The Elica-based applications developed and to be developed within the DALEST project use virtual models of ordinary objects to reveal the beauty of Mathematics. Playing with virtual objects in a way which is almost impossible in reality makes the applications a promising candidate for good learning.

The niche for educational software in the mathematical learning appears to be a long cavern full of miracles waiting to be discovered.

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